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A numerical study of convective heat transfer from a blunt plate at low Reynolds number

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Abstract—The formulation and numerical investigation of convective heat transfer from a two-dimensional blunt rectangular plate is presented. The transformed governing equations in terms of vorticity, stream function and temperature were solved numerically using an efficient finite volume method. A number of interesting features of the flow and heat transfer fields resulting from the leading edge separation are presented in detail. Results including the distribution of friction coefficient and the local Nusselt number were obtained for a range of low Reynolds number. It was found that the physics of the flow and heat transfer changes significantly due to leading edge separation. It was observed that at a Reynolds number, based on plate thickness, of 100 a small separation bubble develops downstream from the leading edge and increases in size in both upstream and downstream extent with an increase in Reynolds number. The leading edge of the bubble reaches the front of the plate at a Reynolds number of 125. The trailing edge continues to move downstream with increasing Reynolds number, reaching a streamwise length of six plate thicknesses at a Reynolds number of 325 and the size of the bubble (or reattachment length) is a linear function of Reynolds number. These findings are consistent with previously published experimental results. The effects of leading edge separation on the local Nusselt number are found to be rather insignificant until the Reynolds number is increased to 225. At $Re = 225$, the local Nusselt number along the plate varies significantly, and a local minimum in the Nusselt number is found inside the separation bubble and a local maximum near reattachment. Also shown for comparison is the variation of local Nusselt number with distance along the plate for the forced convective laminar boundary layer over a thin plate.

1. INTRODUCTION

The term separated flow is generally used to describe the phenomenon which occurs when a steadily flowing liquid or gas, attached to a wall, breaks away and produces a region close to the wall with a reverse flow. This region may occur on smooth surfaces as well as at abrupt changes in the flow geometry, and in both internal and external fluid flows.

The nature of flow separation for a two-dimensional thick rectangular plate is sketched in Fig. 1. As the flow moves from left to right, the boundary layer at the leading edge separates from the wall forming a recirculation bubble within which there is an adverse pressure gradient. Subsequently, the boundary layer reattaches to the wall and approaches the developed state. The mixing in the region between the separation point and the reattachment point plays an important role in the heat and mass transfer processes.

There has been a considerable amount of work carried out to obtain quantitative expressions for heat and mass transfer coefficients on surfaces with boundary layer separation. To date, however, there are no general expressions for calculating heat or mass transfer to or from a separated fluid flowing over a thick rectangular plate.

An adequate theoretical description of heat and mass transfer at solid boundaries in the region of separated and reattached flow is hampered by the lack of a complete theoretical solution of the fluid field. For this reason, investigators have concentrated on the method of dimensional analysis, combined with experimental measurement. While this approach has been eminently successful in producing heat transfer correlations for attached flows, it has not been so for separated flow.

Early separated flow heat transfer studies were primarily concerned with cylinders in cross flow, because of the obvious application to heat exchanger design. Ota and Kon [1] determined experimentally local heat transfer coefficients in the turbulent separated, reattached and redeveloped regions for flow over a horizontal rectangular plate with an aspect ratio of 20. They studied a zero angle of attack and developed an empirical relationship for the Nusselt number at the reattachment point. Under these conditions, they concluded that the flow reattachment occurred at about four plate thicknesses from the leading edge, its position being almost independent of Reynolds number; the position of the maximum Nusselt number and zero shear stress was found to coincide with the reattachment point. Later Ota and co-workers [2–6] and Coney *et al.* [7, 8] published a series of papers on flow separation and reattachment. They reported experimental studies into separated flow, measuring the flow pattern and boundary layer characteristics of flow downstream of reattachment. Furthermore, they measured temperature, velocity and streamwise turbulence intensity within the boundary layer and also

NOMENCLATURE

- D plate thickness
- h_x convective heat transfer coefficient
- H height of upper boundary from the symmetry line
- L convective flux coefficient
- L_d plate length
- L_u distance of upstream boundary to the leading edge
- Nu Nusselt number, hD/k
- Pr Prandtl Number, ν/α
- Re Reynolds number, U_0D/ν
- S_ϕ source term in equation (7)
- T temperature in the boundary layer
- T_0 free stream temperature
- T_w plate temperature
- U dimensionless velocity
- U_0 free stream velocity
- X dimensionless distance, x/D
- x,y Cartesian coordinates
- Y dimensionless distance, y/D.

Greek symbols

- θ dimensionless temperature, $(T-T_0)/(T_w-T_0)$
- Λ diffusion flux coefficient
- Φ variable in equation (7)
- ψ_0 stream function
- ψ dimensionless stream function, ψ_0/U_0D
- ω_0 vorticity function
- ω dimensionless vorticity function, ω_0D/U_0.

Subscripts

- e east side
- n north side
- s south side
- w west side.

heat transfer distributions for various unheated leading edges in their experimental programme. Results were reported for a range of Reynolds number. While their values of heat transfer coefficient cannot be used in a general way, the results add a valuable insight into the mechanism of improved convective heat transfer during flow separation and reattachment.

Experimental data on mass transfer in separated and reattached flow are limited. Kottke et al. [9, 10] investigated mass transfer for flow over a thick plate with various leading edge profiles, including a blunt leading edge at zero angle of attack. In the first paper [9], a plot of Sherwood number against Reynolds number exhibited characteristics similar to those reported by Ota and Kon [1] for the Nusselt number variation. They claimed the existence of two types of separation regions, but the reported Reynolds number range, in which the first separation existed, was so low that it would not be present on blunt plates thicker than 25 mm at velocities grater than 1 m s^{-1}. The second paper by Kottke et al. [10] presented the ratio of local Sherwood number to maximum Sherwood number for a blunt plate at zero angle of attack. The maximum occurred at reattachment. The minimum value was 0.4 and occurred at 0.1 times the reattachment length.

Related numerical studies have also been made for two-dimensional incompressible recirculating flows over bluff bodies. Benodekar et al. [11] studied turbulent flows over surface mounted ribs using a new computational method known as the bounded skew hybrid differencing scheme (BSHD) for incompressible recirculating flows. They compared their numerical results with experimental findings and satisfactory improvements were observed over the previous results. More recently, Djilali and Gartshore [12] and Djilali et al. [13] studied turbulent flow separation around a bluff rectangular plate, using a finite difference procedure. They used two different discretization methods to obtain mean flow field, turbulent kinetic energy and pressure distribution within the separation bubble and they were found to be in good agreement with their experimental data [12]. However, the range of interest in many heat exchanger applications extends to much lower Reynolds number than previously investigated. Lane and Loehrke [14]

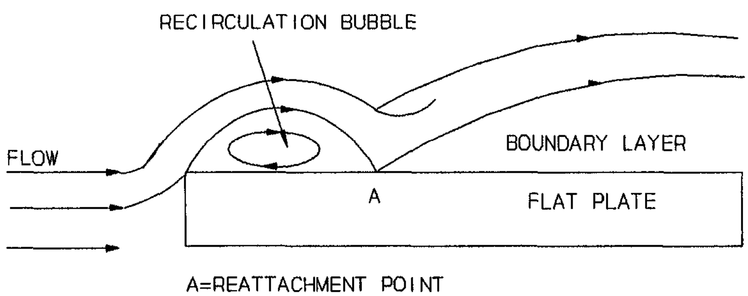

Fig. 1. Flow pattern over a blunt flat plate.

investigated experimentally the flow over a blunt plate at low Reynolds number aligned parallel to the free stream. They visualized using dye tracers. They observed a leading edge separation bubble forming at a Reynolds number based on plate thickness of 100 and growing in size with increasing Reynolds number, reaching a maximum streamwise steady recirculation length of 6.5 times the plate thickness at $Re = 325$. At higher Reynolds number the separated shear layer became unsteady and turbulent in nature.

As far as the authors are aware, detailed theoretical work on heat transfer characteristics of laminar boundary layer separation seem to be limited and the enhancement mechanism due to recirculation in the laminar separated flow is still not fully understood. The theoretical work reported here was performed to provide a direct check of these predictions and to fill the gap between laminar and turbulent measurement. For this reason, the two-dimensional, steady, laminar incompressible Navier–Stokes equations coupled with the energy equation for separated and reattached flow around a bluff rectangular plate are solved numerically in terms of vorticity and stream function using a finite volume method. Velocity profiles and temperature profiles within the boundary layer are obtained for a range of low Reynolds number, $70 < Re < 325$, in the separation region and downstream of reattachment. Also determined for these regions is the distribution of friction coefficient and local Nusselt number with distance along the plate for various Reynolds number. Also shown for comparison is the variation of local Nusselt number with distance along the plate for the forced convective laminar boundary layer over a thin plate.

2. FORMULATION OF THE MODEL

2.1. *Governing equations*

For an steady two-dimensional laminar flow, the non-dimensional governing equations are:
vorticity transport equations:

$$\left(\frac{\partial^2\psi}{\partial X^2}+\frac{\partial^2\psi}{\partial Y^2}\right)+\omega = 0 \tag{1}$$

$$\frac{\partial}{\partial X}\left(\omega\frac{\partial\psi}{\partial Y}\right)-\frac{\partial}{\partial Y}\left(\omega\frac{\partial\psi}{\partial X}\right)-\frac{1}{Re_D}\left(\frac{\partial^2\omega}{\partial X^2}+\frac{\partial^2\omega}{\partial Y^2}\right) = 0 \tag{2}$$

energy equation:

$$\frac{\partial}{\partial x}\left(\theta\frac{\partial\psi}{\partial Y}\right)-\frac{\partial}{\partial Y}\left(\theta\frac{\partial\psi}{\partial X}\right)-\frac{1}{Re_D\,Pr}\left(\frac{\partial^2\theta}{\partial X^2}+\frac{\partial^2\theta}{\partial Y^2}\right) = 0. \tag{3}$$

The computational domain and the boundary conditions for the plate are shown in Fig. 2. At the upstream position, AF, uniform flow condition is assumed for all the flow parameters, i.e.

$$\begin{cases} X = 0 & 0 \leqslant Y \leqslant H/D \\ U = 1 & \omega = 0 \\ V = 0 & \psi = Y. \end{cases} \tag{4}$$

This boundary is located by trial and error and it is far enough from the plate for the flow at this plane to be unaffected by the presence of the plate.

A zero streamwise gradient condition, $\partial(\ )/\partial X = 0$, is applied across the outflow boundary DE. Although this boundary condition is strictly valid only when the flow is fully developed, its use in other flow conditions is also permissible for computational convenience provided that the outlet boundary is located in a region where the flow is in the downstream direction and sufficiently far downstream from the region of interest.

Along the symmetry axis AB, a zero cross-stream gradient condition $\partial()/\partial Y = 0$ is specified with $V = 0$, $\psi = 0$ and $\omega = 0$. No slip boundary conditions are imposed along boundaries BC and CD. However, over these boundaries, the following formula is used for vorticity as

$$\begin{cases} \omega_p = -\left[\dfrac{3(\psi_{np}-\psi_p)}{n_{np}^2}+\dfrac{\omega_{np}}{2}\right] \\ \psi_p = 0 \\ \theta_p = 1 \end{cases} \tag{5}$$

where np pertains to a node which lies adjacent to a boundary node p and on a normal to the boundary. It should be noted that in the external flow condition, the location of the boundary condition that is satisfied at infinity, i.e. FE, is very important and is obtained by trial and error for a distance sufficiently far from the plate

$$\begin{cases} Y = H/D & L_u/D < X < L_d/D \\ \psi = H/D & \omega = 0 \quad \theta = 0. \end{cases} \tag{6}$$

2.2. *Numerical analysis*

A finite volume formulation is used to derive algebraic approximations to the partial differential equations. Figure 3 shows a control volume around a typical grid point P.

The usual finite volume process of integrating equations (1)–(3) over the control volume at P involves the introduction of suitable centred difference approximations for the diffusion terms as

$$L_w\Phi_w - L_e\Phi_e + L_s\Phi_s - L_n\Phi_n + \Lambda_e(\Phi_E-\Phi_P) - \Lambda_W(\Phi_P-\Phi_W)+\Lambda_n(\Phi_N-\Phi_P) - \Lambda_s(\Phi_P-\Phi_S) - S_\Phi\Delta x\Delta y = 0. \tag{7}$$

The BSHD scheme is used as described in refs. [11, 15] to approximate the convective fluxes, i.e. the first four terms on the left-hand side of equation (7).

The BSHD scheme takes into account the local

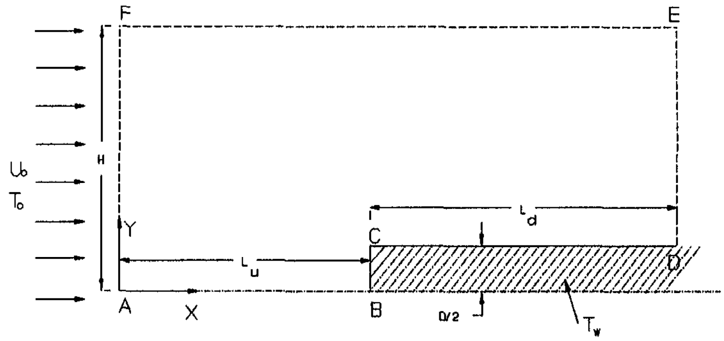

Fig. 2. The computational domain.

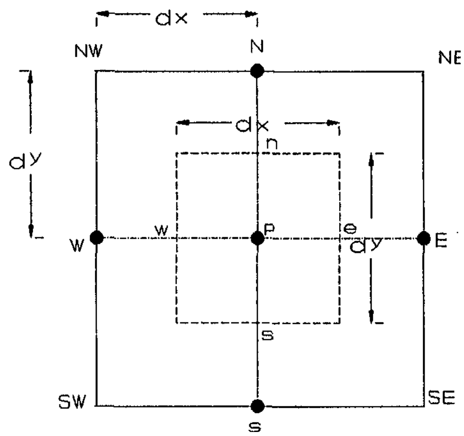

Fig. 3. The finite volume grid representation.

direction of the flow, thereby greatly reducing the 'skewness error' which, in two-dimensional flow, is often the largest contribution to false diffusion associated with the upwinding of convective terms which results from non-alignment of the coordinate grid with the flow direction.

A staggered non-uniform grid arrangement is used in the present computations. The set of difference equations is solved iteratively by point successive over-relaxation. It is important to note that the protruding corner C is a singularity and vorticity is evaluated according to the implicit procedure for constant y at the wall.

3. RESULTS AND DISCUSSION

3.1. *Velocity profiles*

Shown in Fig. 4(a) is a typical velocity profile in the boundary layer for $Re = 325$ (based on plate thickness) in the separation region. The mean velocity, U, and the perpendicular distance, y, were normalized by the free stream velocity, U_0, and plate thickness, D, respectively. Consecutive velocity profiles in the separation region, Fig. 4(b), and after reattachment, Fig. 4(c), were plotted at increasing distance from the leading edge.

One of the most interesting features of the velocity profiles in the separation region, Fig. 4(b), is the extremely rapid acceleration of the flow near the wall. A similar result was also noted by Ota and Itaska [2], however their experiments were conducted for low turbulent flow. This acceleration is more rapid in the region near the leading edge and in the separated region.

The velocity profiles in the vicinity of separation indicate that a region of relatively stagnant flow exists near the wall, characteristic of separation. The velocity profiles in the separation region near the wall also shows a region of reverse flow.

The boundary layer reattaches at about six plate thicknesses downstream, which is almost consistent with the reattachment length predicted by Lane and Loehrke [14]. It then redevelops and attains profiles similar to those for laminar flow over a sharp-edged flat plate, Fig. 4(c). The boundary layer thickness in the separation region is equal to the plate thickness, while in the redeveloped region downstream of the reattachment point it is approximately equal to one and a half times the plate thickness.

The velocity vectors at various locations along the plate for $Re = 325$ in the separation region, Fig. 5(a), and after reattachment and the redeveloped region, Fig. 5(b) and (c), are shown. In Fig. 5(a) the reverse flow within the recirculation bubble and at the reattachment point can be seen clearly.

3.2. *Temperature profiles*

The mean temperature distributions in the boundary layer for $Re = 325$ are illustrated in Fig. 6(a)–(c). In the separation region, the temperature distribution shows some deviation from other regions. In the developed flow region, the thermal boundary layer thickness is about the same as the plate thickness.

3.3. *Variations of reattachment length and friction coefficient*

The variation of recirculation length with Reynolds number is shown in Fig. 7. From the velocity profiles obtained in ref. [15], it was observed that at $Re = 100$, a small bubble of size $x_r/D = 0.4$ developed at $x/D = 0.3$ downstream from the leading edge and extended to a distance of $x/D = 0.7$. As the Reynolds number is increased the bubble grows both upstream

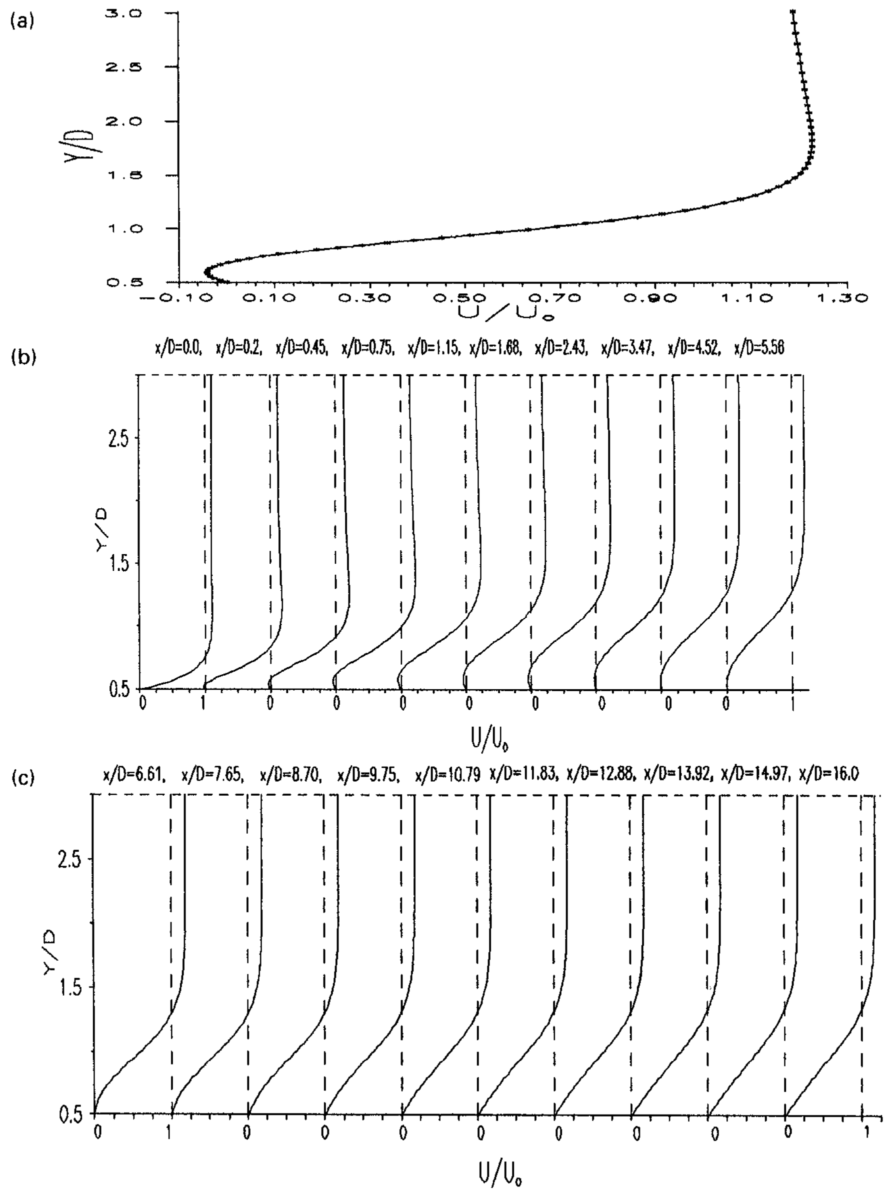

Fig. 4. (a) Typical velocity profile in the separation region, $Re = 325$. (b) Velocity profiles in the separation region, $Re = 325$. (c) Velocity profiles in the redeveloped region, $Re = 325$.

and downstream. The leading edge of the bubble reaches the front of the plate at $Re = 125$. The trailing edge continues to move downstream with increasing Reynolds number to $x/D = 6.0$ at $Re = 325$. The following correlation for reattachment length in terms of Reynolds number was obtained using a linear least-squares fitting technique:

$$\frac{X_r}{D} = -1.9 + 0.025 Re_D. \quad (8)$$

The variation of friction coefficient, C_f, along the plate is shown in Fig. 8. For $Re = 325$, the value of C_f in the neighbourhood of the reattachment point is quite small. It then increases rapidly downstream of the reattachment point. Subsequently it becomes constant downstream. The constant is attained at $x/D = 14.0$ and its value is about 0.007.

3.4. *Variation of local Nusselt number*

The variation of Nusselt number, Nu_D, with non-dimensional distance, x/D, along the plate for a number of values of Re in the range 75–325 (based on plate thickness) is plotted in Fig. 9. These results show that for $Re < 225$ the separation bubble does not affect the local Nusselt number appreciably and the variation of local Nusselt number is similar to that predicted by the conventional laminar boundary layer expression for a thin plate. However, for $Re > 225$, the local Nusselt number decreases initially from the

(a)

(b)

(c)

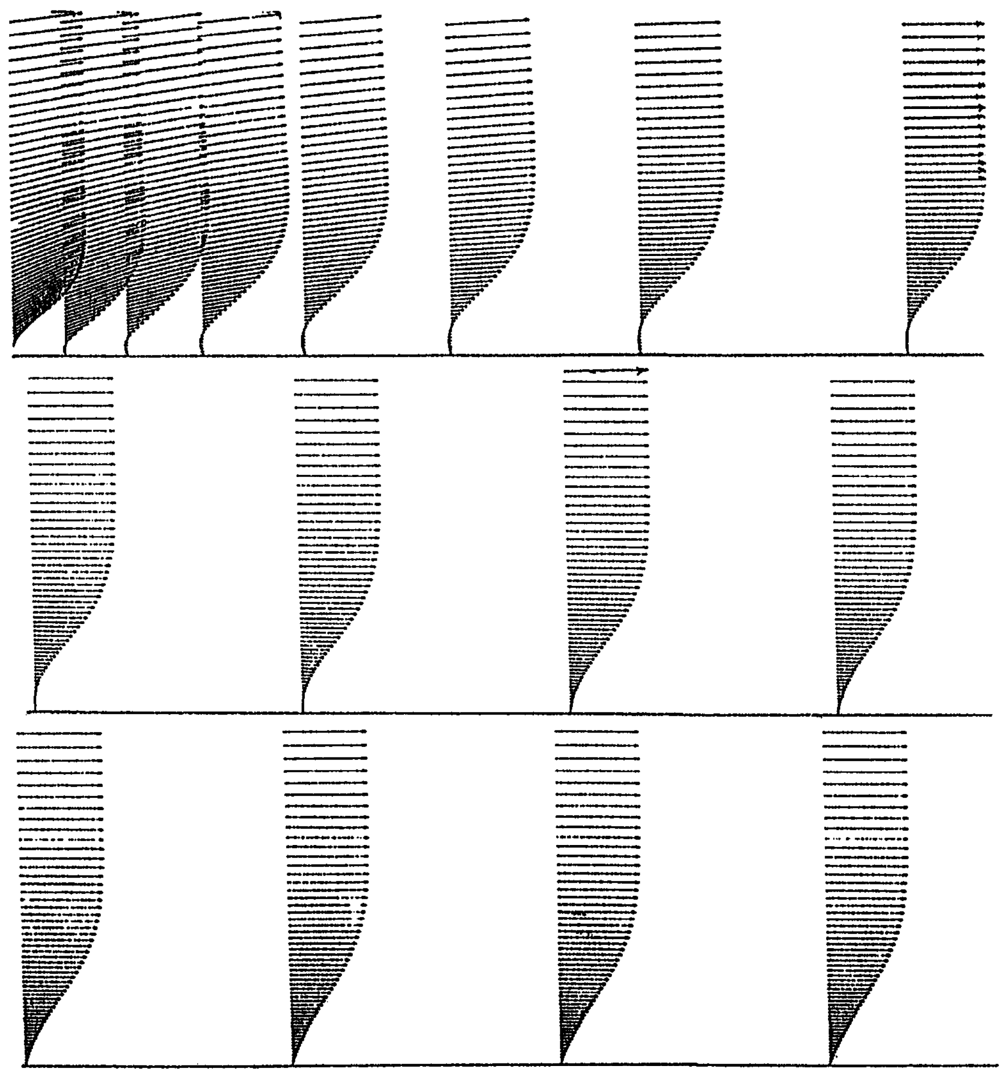

Fig. 5. (a) Velocity vectors in the separation region, $Re = 325$. (b, c) Velocity vectors in the redeveloped region, $Re = 325$.

leading edge to about 2.5 times the plate thickness before starting to increase. This characteristic is attributed to a strong shear layer existing at the leading edge. A similar trend was also observed in the experimental work carried out by Coney *et al.* [8] and Ota and Kon [1]. Since most of the resistance to heat transfer is found in the sublayers, this decrease can also be due to their rapid readjustment.

Ota and Kon [1] defined the position of reattachment as the point on the plate at which the local Nusselt number has a maximum in turbulent separated flow. In Fig. 9, the maximum local Nusselt number is shown to occur at about eight times the plate thickness downstream from the leading edge, whereas the reattachment length predicted by the velocity profiles shown in Fig. 4(b) is about six times the plate thickness. It then quickly decreases after reattachment to what appears to be a final near constant state approaching slightly higher values than those for a laminar boundary layer on a thin flat plate without separation, Fig. 10.

The ratio of minimum to maximum Nusselt number in the separated region is about 0.8; this ratio decreases slightly with decreasing Reynolds number. The minimum occurs at about 0.3 times the distance to reattachment for Reynolds numbers greater than 225; however, the results of heat transfer for turbulent separated flow investigated by Coney *et al.* [8] and the mass transfer investigated by Kottke *et al.* [10] showed that the ratio of minimum to maximum is about 0.4 and the minimum occurred at 0.1 times the distance to reattachment.

In order to understand the mechanisms leading to the augmentation heat transfer when flow is made to separate and reattach, it is appropriate to refer to the velocity profiles discussed previously, Fig. 4(b). There, it was shown that the velocity inside the separation bubble was quite high, hence there ought to be a

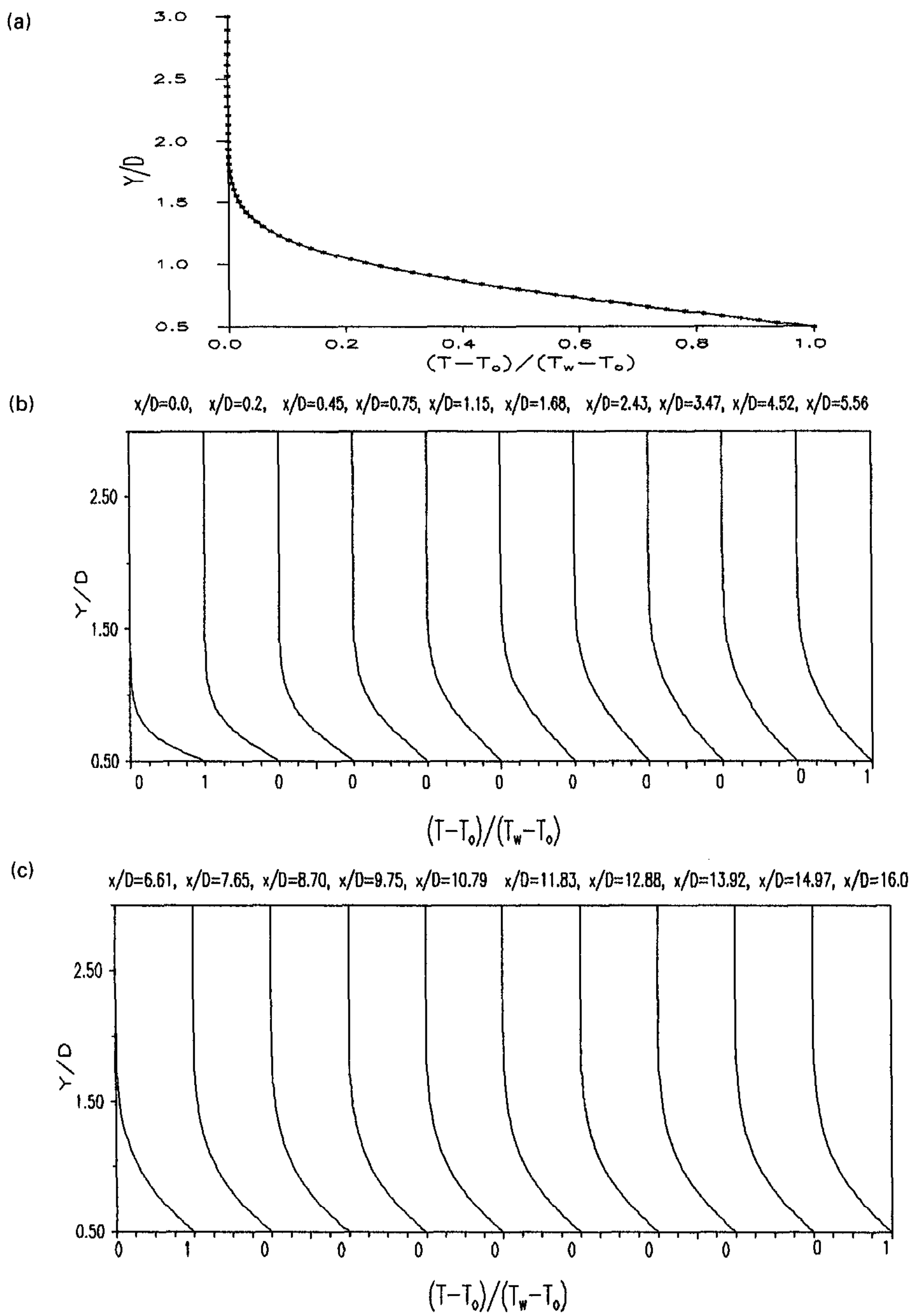

Fig. 6. (a) Typical temperature profile in the separation region, $Re = 325$. (b) Temperature profiles in the separation region, $Re = 325$. (c) Temperature profiles in the redeveloped region, $Re = 325$.

corresponding high heat transfer rate inside the bubble. However, the local minimum Nusselt number found inside the bubble indicates that the recirculating flow is of lower velocity.

The local maximum in the region of reattachment may be interpreted as being due to the vortex action drawing cooler fluid towards the surface immediately after separation. The fluid drawn close to the surface in the region of reattachment is relatively cool, having flowed over the separation bubble away from the heated surface. The relatively large temperature difference between the plate surface and this cooler fluid accounts for the high heat transfer rates observed at the reattachment point.

Increasing the Reynolds number is seen to shift slightly the mean reattachment point away from the leading edge and to increase slightly the maximum heat transfer coefficient. At Reynolds number greater than 225, there is no significant movement of the position of minimum and maximum heat transfer coefficient.

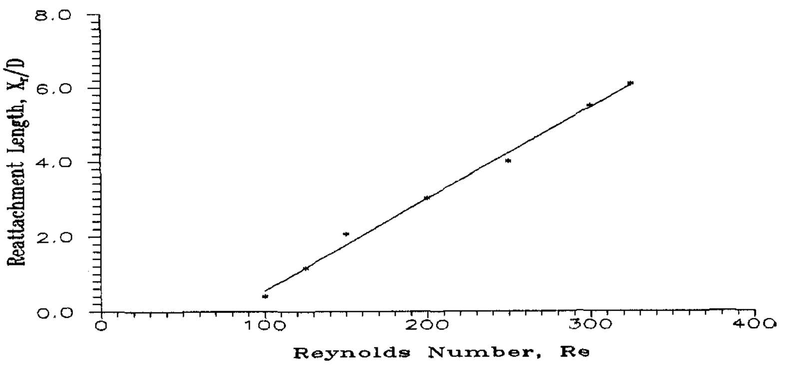

Fig. 7. Variation of recirculation length with Reynolds number.

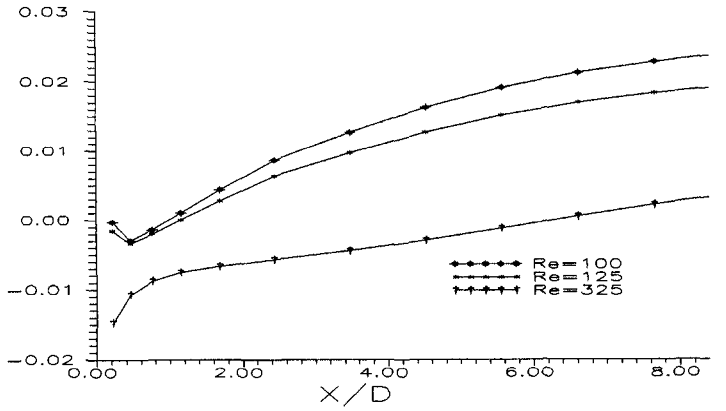

Fig. 8. Variation of friction coefficient, C_f, along the plate at various Reynolds numbers.

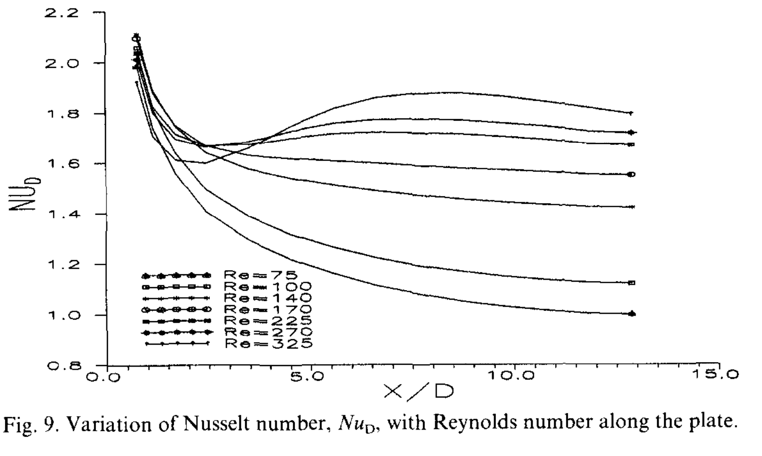

Fig. 9. Variation of Nusselt number, Nu_D, with Reynolds number along the plate.

4. CONCLUSIONS

The heat transfer characteristics in laminar separated, reattached and redeveloped flow over a plate of finite thickness has been investigated numerically. It has been found that the friction coefficient and heat transfer are affected when the flow is made to separate and reattach. The velocity profiles obtained in ref. [15] showed that for $Re = 100$ a small separation bubble exists near the leading edge and grows in size with increasing Reynolds number. The recirculation length varies linearly with Reynolds number. The bubble reaches a maximum length of about six times the plate thickness at a Reynolds number of $Re = 325$. The variation of friction coefficient along the plate showed that in the neighbourhood of the reattachment point the friction coefficient is quite small. It then increases rapidly downstream of the reattachment and subsequently reaches a constant downstream.

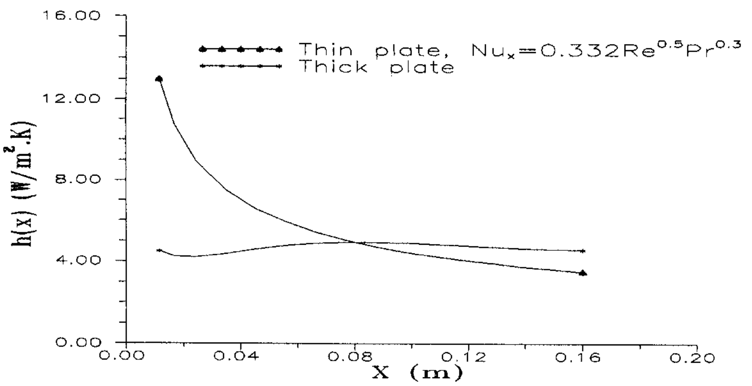

Fig. 10. Comparison of the local heat transfer coefficient for both thin and thick plates.

The effects of leading edge separation on the local Nusselt number is rather insignificant until the Reynolds number is increased to $Re = 225$. For $Re = 225$, the first local maximum appears and for $225 \leqslant Re \leqslant 325$, the heat transfer characteristics along the plate surface vary significantly, and a local minimum in the Nusselt number is found inside the separation bubble and a local maximum near reattachment. The maximum Nusselt number occurs at about $x/D = 6.0$ for $Re = 225$ and grows in size with increasing Reynolds number to $x/D = 8.0$ for $Re = 325$. The ratio of minimum to maximum Nusselt number is about 0.8, the minimum occurring at about $x/D = 2.5$, 0.4 times the reattachment length obtained from the velocity profiles. This ratio decreases with an increase in Reynolds number.

The local heat transfer coefficient occurring over the plate with boundary layer separation was also compared with that for a laminar boundary layer over a thin flat plate at $Re = 325$. It was found that for $x/D < 8$, the local heat transfer coefficient was lower due to leading edge separation; however, for $x/D > 8$, higher values are obtained.

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